Collective-Action Games

Edicson Luna

August 15, 2025

Collective-Action Games with Two Players

You and a neighboring farmer both benefit from an irrigation and flood-control project.

- You can build it together or alone.
- Once built, both of you benefit—regardless of who contributed.
- Each player is tempted to free ride on the other's effort.

Key problem: Strategic interaction makes cooperation difficult when benefits are shared but costs are private.

From Two to Many Players: Collective-Action Problems

Real-life examples:

- Organizing student groups or local events.
- Rush-hour commuting or ride-sharing.
- Climate change mitigation or public protests.

The dilemma:

- Often too few contribute, or too many overuse a common good.
- Outcomes are often inefficient for everyone.

Core idea:

- The socially optimal outcome is not always a Nash equilibrium.
- Private incentives may lead to a suboptimal result.

PD and Chicken

	Neighbor	
You	**Build**	**Not**
Build	(4, 4)	$(-1, 6)$
Not	$(6, -1)$	(0, 0)

PD: Socially optimal outcome (Build, Build) is not achieved.

	Neighbor	
You	**Build**	**Not**
Build	(5, 5)	(2, 6)
Not	(6, 2)	(0, 0)

Chicken: one builds, the other shirks. But who should build?

Assurance and Collective Inaction

	Neighbor	
You	**Build**	**Not**
Build	(4, 4)	(-4, 3)
Not	(3, -4)	(0, 0)

(Build, Build) is preferred by both players. No conflict over which equilibrium is better.

Collective inaction

- Some collective-action problems involve inaction, where overuse harms everyone (e.g., traffic, fishing).
- Failure to coordinate leads to the "tragedy of the commons": When individuals overuse a shared resource for personal gain, the resource becomes depleted.
- Solutions often involve changing incentives—such as assigning clear ownership or property rights (e.g., de Soto), government regulation, or community-based monitoring and enforcement.

Tragedy of the commons

Large Groups: PD

Problem Setup:

- A village of $N = 100$ farmers must decide whether to participate in building a shared irrigation project.
- Participation increases productivity for everyone, even for those who don't participate.
- Each farmer faces a tradeoff: contribute effort (and pay a cost), or free ride and still benefit.

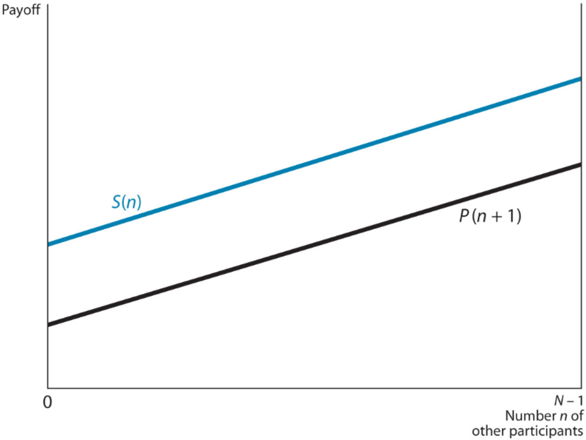
Payoff Functions (example):

- Benefit from participating with n others: $P(n+1) = 2(n+1)$
- Benefit from shirking: $S(n) = 2n + 4$

Key Insight:

- Since $S(n) > P(n+1)$ for all n, each farmer prefers to shirk.
- Nash equilibrium: $n = 0$ (no one participates).
- Socially optimal outcome: $n = N$ (everyone participates).

Large Groups: PD



This mirrors the logic of the Prisoners' Dilemma (shirking is a dominant strategy), but in a large group.

Large Groups: Chicken Game

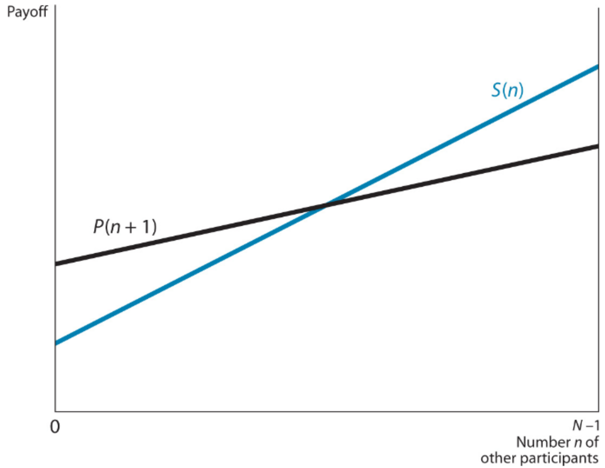
- Participate: $P(n) = 4n + 36 \quad \Rightarrow \quad P(n+1) = 4n + 40$
- Shirk: $S(n) = 5n$

The idea here is...

- Participation yields a fixed baseline payoff (36) and increases slowly with more contributors.
- Shirking gives no baseline payoff, but increases faster as others contribute.
- The incentive to shirk grows faster than the reward for participating.

<u>Core Idea:</u> When few people are helping, it's worth joining in. When many already contribute, it's tempting to free ride.

Large Groups: Chicken Game



Externalities

An externality occurs when an individual's action imposes a cost or benefit on others that is not reflected in their private decision.

Examples:

- **Negative externality:** More cars on the road $\rightarrow$ slower travel time for everyone.
- **Positive externality:** A neighbor plants trees $\rightarrow$ cleaner air and shade for others.

When private incentives don't align with group outcomes, society faces problems like underprovision of public goods or overuse of shared resources.

Negative externality: Commuting

- 8,000 commuters choose between two routes: local roads (S) and expressway (P).
- The payoffs are represented as how much the commute time is less than 1 hour.
  - **Local roads (S):** Always takes 45 minutes $\rightarrow$ payoff: $S(n) = 15$.
  - **Expressway (P):** Takes 15 minutes when empty; increases by 0.005 minutes per user.
  - Payoff for expressway users: $P(n) = 60 - (15 + 0.005n) = 45 - 0.005n$.

Negative externality: Commuting

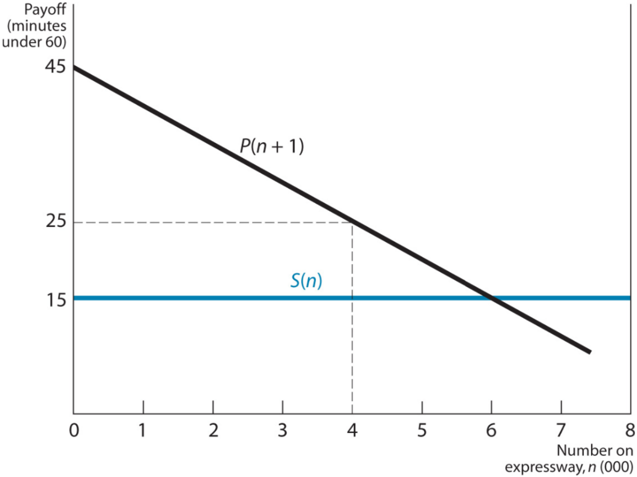


FIGURE 11.9 Commuting Route-Choice Game

Negative externality: Commuting

Example:

- Suppose $n = 4{,}000$ drivers use the expressway $\rightarrow$ commute time $= 35$ minutes.
- $P(4000) = 25$; $S = 15$ $\rightarrow$ One local-road user has incentive to switch.
- $P(4001) = 24.995 > S = 15$ $\rightarrow$ They will switch

Private incentive: Switch because your individual commute improves.

Negative externality: Commuting

- Marginal private gain:

$$P(n+1) - S = 24.995 - 15 = 9.995 \text{ minutes saved}$$

- Marginal spillover effect:
  - Your switch delays 4,000 others by 0.005 minutes each $\rightarrow$ total loss $=$ 20 minutes
- Marginal social gain:

$$\text{Private gain} - \text{External cost} = 9.995 - 20 = -10.005$$

Your action benefits you but harms society. This is a perfect example of a negative externality.

Private decisions don't internalize spillover costs and total efficiency drops.

Negative externality: Commuting

$$P(n) = S \Rightarrow 45 - 0.005n = 15 \Rightarrow n = 6000$$

At $n = 6000$:

- Expressway payoff: $P(6000) = 45 - 0.005 \cdot 6000 = 15$
- Local roads payoff: $S = 15$
- All commuters are indifferent between routes.

- If one expressway user switches to local roads: payoff $= 15$ (same).
- If one local-road user switches to expressway: $P(6001) = 14.995 \rightarrow$ worse off.

No one has a profitable deviation. Nash equilibrium holds.

This NE is defined by having exactly 6000 users on the expressway, not by which commuters those are.

Negative externality: Commuting

At $n = 5999$:

- Expressway payoff: $P(5999) = 45 - 0.005 \cdot 5999 = 15.005$
- Local roads payoff: $S = 15$

Deviations:

- Expressway users: prefer to stay $(15.005 > 15)$
- Local-road users:
  - ⋆ If they switch: $P(6000) = 15$, which is equal to S.
  - ⋆ No strict gain.

No one has a profitable deviation. Nash equilibrium holds.

This NE is determined by having 5999 users on the expressway, regardless of who they are.

Positive externality: Technology choice

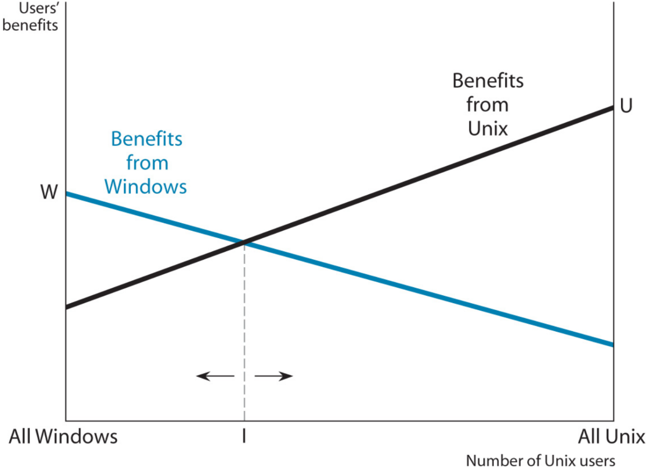
Each person chooses between two operating systems: Windows or Unix (e.g., Linux).
The benefit of using an OS increases with the number of other users:

- More users $\rightarrow$ more software, fewer bugs, more help.
- These are positive externalities. We can also call it network effects (in this case).

Payoffs:

- Unix benefits rise with number of Unix users.
- Windows benefits fall as Unix becomes more popular.

Positive externality: Technology choice



Positive externality: Technology choice

- The intersection point I: where benefits of Windows and Unix are equal.
- Left of I: Better to choose Windows.
- Right of I: Better to choose Unix.

As a result, we get

- Two NEs: all-Windows or all-Unix.
- The equilibrium depends on where the game starts.
- Small shifts near I can lead to large cascades.

Positive externality: Technology choice

What if Unix is better for society?

- Even if Unix is superior, early popularity of Windows may lead society to be locked in to a worse equilibrium.
- Individual users lack incentive to switch unless a large group does so.

Broader Examples:

- QWERTY vs. Dvorak keyboards
- Gasoline vs. electric cars
- VHS vs. Betamax

Sometimes society needs a coordinated push or public intervention to reach a better outcome.
Good ideas alone are not enough—coordination matters.